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Nonvanishing anomalous Hall conductivity in spin-polarized two-dimensional electron gas with Rashba spin–orbit interaction

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Abstract
Based on Kubo's linear response theory, we discuss the anomalous Hall effect (AHE) in a two-dimensional electron gas (2DEG) with Rashba spin–orbit coupling (SOC) subjected to a homogeneous out-of-plane magnetization, by taking into account the coupling between the anisotropic magnetic impurities and itinerant electrons. For a weak, short-ranged impurity potential, in the limit of $\varepsilon_{\mathrm{f}} \gg \hbar/\tau$, Δ, the self-energy is calculated in the Born approximation, and the vertex correction to the Hall conductivity is taken into account by the ladder approximation. Then the anomalous Hall conductivity in the steady state ($\omega = 0$) is nonvanishing at zero temperature in the presence of the magnetic impurities, which is different from the nonmagnetic impurities condition.

1. Introduction

Spintronics has become a fast developing area using the electron spin degrees of freedom, rather than the conventional electron charge in electronic devices, and has attracted the attention of many scientists in the last several years [1–4]. The spin–orbit coupling (SOC), which couples the electron momentum and spin, can serve as a spin-charge mediator. Also, the study of SOC in mesoscopic systems has been the subject of extensive research in recent years, for instance the spin Hall effect (SHE) in normal semiconductors and the anomalous Hall effect (AHE) in ferromagnets.

The theoretical discussion of the AHE has a long, controversial history. As early as in the 1950s, in the pioneering work by Karplus and Luttinger [5], it was first pointed out that the AHE in ferromagnets results from the interplay of the exchange field and spin–orbit coupling. Also, soon after, as was developed by Smit [6] and Berger *et al* [7], the so-called extrinsic Hall contribution was attributed to the skew and side-jump scattering at impurities. On the other hand, the so-called intrinsic Hall contribution was attributed to the Berry phase in the absence or presence of impurities [8, 10] and, in recent years, it has been developed on the basis of the Kubo–Streda linear response formalism or semiclassical Boltzmann transport theory [9, 11, 12].

Another interesting phenomenon arising from the SOC is the SHE, which describes the way that, in a normal semiconductor, when an external electric field is applied in the plane, a persistent spin current appears in the transverse direction without any net charge current in this direction. Also, this has developed rapidly due to its potential applications in the last several years, including theoretical discussions and experimental observations. Analogously to the AHE, two different mechanisms of SHE are proposed: the extrinsic SHE [13–16], which is associated with impurity scattering, such as skew scattering and the side-jump process. On the other hand, intrinsic SHE was proposed by Murakami *et al* [19] in a p-type semiconductor for a Luttinger [17] Hamiltonian and by Sinova *et al* [20] in a two-dimensional electron gas (2DEG) with Rashba [18] SOC in the clean limit. In addition to the theoretical development, on the experimental side, two typical experiments have been reported by Kato [21] and Wunderlich [22], explained as the extrinsic and intrinsic SHE, respectively.

But unfortunately, in the presence of impurities, when vertex correction is taken into account, the spin Hall conductivity is vanishing for the 2DEG with Rashba SOC in the dc condition [23]. Similarly to the conclusion obtained by Smit [6] for the anomalous Hall conductivity in ferromagnets, and from then on, the effect of impurities has been investigated extensively, including isotropic [24] and anisotropic magnetic impurities [25]. They found that, for both kinds of magnetic impurities, the spin Hall conductivity is nonzero for the Rashba SOC system. In addition, in Inoue *et al*'s work [24], the anomalous Hall conductivity has also been discussed in the presence of isotropic but spin-dependent impurities, and it was reported that, only when the self-energy is spin-dependent, the anomalous Hall conductivity is nonzero, otherwise the Hall conductivity is vanishing. Due to the resemblance between the AHE and SHE, one may view the SHE as the zero magnetization limit of the AHE [24], and now, with the SHE surviving in anisotropic magnetic impurities, can the AHE exist likewise? This leaves an opportunity for nonvanishing anomalous Hall conductivity and motivates us to have a further discussion about the AHE in the present paper.

In this paper, we calculate the anomalous Hall conductivity for a two-dimensional magnetically disordered Rashba-electron gas, subjected to a uniform magnetization perpendicular to the plane. Also, the so-called XXZ-type anisotropic interaction between the magnetic impurities and the electron is applied. In the limit of large Fermi energy ($\varepsilon_{\mathrm{f}} \gg \hbar/\tau$, Δ), based on Kubo's linear response theory, the anomalous Hall conductivity is nonvanishing even when the vertex correction is taken into account, which is obtained within the Born approximation by a ladder diagram, which is different from the conditions of nonmagnetic and isotropic spin-dependent impurities [24]. In addition, for the case that we discuss here, the anomalous Hall conductivity is anisotropy dependent.

2. The model with magnetic impurities

Let us consider a two-dimensional electron gas with Rashba spin–orbit coupling in the presence of impurities, and the Hamiltonian can be written as:

$$H = H_0 + V_{\mathrm{m}} \tag{1}$$

where H_0 is the unperturbed Hamiltonian for the Rashba 2DEG, subjected to a uniform magnetization perpendicular to the plane, consisting of noninteracting electrons of mass m:

$$H_0 = \frac{\hbar^2 k^2}{2m} + \alpha(k_y\sigma_x - k_x\sigma_y) + M\sigma_z. \tag{2}$$

Here $\mathbf{k}$ is the wavevector, confined in the xy-plane with $\mathbf{k} = (k_x, k_y) = k(\cos\varphi, \sin\varphi)$. $\widehat{\sigma_i}$ ($i = x, y, z$) are the usual Pauli matrices, α is the spin–orbit coupling constant, which is tunable by an external electric gate, and M is the strength of the out-of-plane magnetization, which is assumed to have energy units for simplification. For a given $\mathbf{k}$, the eigenfunctions of the Hamiltonian H_0 can be expressed as

$$|\mathbf{k},\lambda\rangle = \begin{pmatrix} \dfrac{\mathrm{i}\lambda\alpha k \mathrm{e}^{-\mathrm{i}\varphi}}{\sqrt{(\sqrt{M^2+\alpha^2k^2}-\lambda M)^2+\alpha^2k^2}} \\ \dfrac{\sqrt{M^2+\alpha^2k^2}-\lambda M}{\sqrt{(\sqrt{M^2+\alpha^2k^2}-\lambda M)^2+\alpha^2k^2}} \end{pmatrix} \tag{3}$$

where $\lambda = \pm 1$, and the corresponding eigenvalues of H_0 are

$$\varepsilon_\lambda = \frac{\hbar^2k^2}{2m} + \lambda\sqrt{M^2+\alpha^2k^2}. \tag{4}$$

Also, the carrier's velocity operator reads $\mathbf{v} = \mathrm{i}[H_0, \mathbf{r}]/\hbar$, with $\mathbf{r}$ being the position operator, or they can be described in terms of components,

$$v_x = \frac{\hbar k_x}{m} - \frac{\alpha}{\hbar}\sigma_y; \tag{5}$$

$$v_y = \frac{\hbar k_y}{m} + \frac{\alpha}{\hbar}\sigma_x. \tag{6}$$

V_{m} is the random potential caused by impurities. Here the random potential that we considered is supposed to be weak and short-ranged, but anisotropic and spin-dependent, described as the XXZ-type interaction between the magnetic impurities and the electron spin:

$$V_{\mathrm{m}} = \sum_{i=1}^{N}\int \mathrm{d}\mathbf{r}^2 u\delta(\mathbf{r}-\mathbf{R}_i)\widehat{\psi}^{+}(\mathbf{r}) \times \begin{pmatrix} \gamma\cos\theta_i & \sin\theta_i \mathrm{e}^{-\mathrm{i}\phi_i} \\ \sin\theta_i\mathrm{e}^{\mathrm{i}\phi_i} & -\gamma\cos\theta_i \end{pmatrix}\widehat{\psi}(\mathbf{r}) \tag{7}$$

where $\mathbf{R}_i$ is the position of the impurity, and (θ_i, ϕ_i) denotes the orientation of the impurity. N is the number of impurities, and u and γ describe the strength and the anisotropy of the coupling between the itinerant electrons and the magnetic impurities, respectively. When $\gamma = 1$, it represents the isotropic magnetic impurity, and has been discussed in the previous literature of [24] for the spin Hall conductivity in the absence of magnetization. For simplicity, in this work we assume that the distribution of impurities is homogeneous and that the magnetic orientations of the impurities induced by their surrounding local spherical symmetry being broken could still be considered random when the magnetization M is no stronger than each coercive field of local magnetic impurity.

3. Anomalous Hall conductivity

3.1. Self-energy and averaged Green's functions

Before the calculation of the anomalous Hall conductivity, we should work out the self-energy of the whole unperturbed system first. In the absence of impurity scattering, the Green's function is described by the Hamiltonian H_0 in equation (2): $G_0 = (\varepsilon - H_0)^{-1}$, and the retarded and advanced Green's functions can be expressed as

$$G_0^{r/a}(\varepsilon,\mathbf{k}) = \frac{1}{2}\sum_{\lambda=\pm1}\frac{1}{\varepsilon-\varepsilon_\lambda \pm \mathrm{i}\eta} \times \left(1+\lambda\frac{\alpha(k_y\sigma_x-k_x\sigma_y)+M\sigma_z}{\sqrt{M^2+\alpha^2k^2}}\right), \tag{8}$$

respectively, for $\eta \to 0^+$.

Due to the presence of an impurity potential, the Green's functions should be modified. Taking into account the weak, short-ranged, anisotropic, and homogeneous magnetic impurities in the Born approximation, the corresponding self-energy of the electrons for the averaged retarded and advanced Green's functions are [26]

$$\Sigma^{r/a} = \frac{n_{\mathrm{i}}u^2}{V}\sum_{\mathbf{k}}\int\frac{1}{4\pi}\mathrm{d}\theta\,\mathrm{d}\phi\,\sin\theta\begin{pmatrix}\gamma\cos\theta & \sin\theta\mathrm{e}^{-\mathrm{i}\phi} \\ \sin\theta\mathrm{e}^{\mathrm{i}\phi} & -\gamma\cos\theta\end{pmatrix} \times G_0^{r/a}(\varepsilon,\mathbf{k})\begin{pmatrix}\gamma\cos\theta & \sin\theta\mathrm{e}^{-\mathrm{i}\phi} \\ \sin\theta\mathrm{e}^{\mathrm{i}\phi} & -\gamma\cos\theta\end{pmatrix} \tag{9}$$

where V is the area of the 2DEG, and n_i denotes the impurity concentration. After some tedious but straightforward calculation, we can get that the self-energy is diagonal in the space of momentum $\mathbf{k}$, and is independent of momentum and spin. The imaginary components of the two Green's functions are:

$$\mathrm{Im}\,\Sigma^{r/a}(\varepsilon) = \mp\frac{\hbar}{2\tau} \tag{10}$$

where τ denotes the momentum relaxation time, $\hbar/\tau = n_\mathrm{i}u^2 2\pi\nu(\gamma^2+2)/3$, and $\nu = m/2\pi\hbar^2$ is the two-dimensional density of states, which is the same as in the absence of magnetization [25, 27]. Taking into account the contribution from the imaginary component of the self-energy, and neglecting the real part, the two averaged retarded and advanced Green's functions are

$$G^{r/a}(\varepsilon,\mathbf{k}) = \frac{1}{2}\sum_{\lambda=\pm}\frac{1}{\varepsilon-\varepsilon_\lambda \pm \mathrm{i}\frac{\hbar}{2\tau}} \times\left(1+\lambda\frac{\alpha(k_y\sigma_x-k_x\sigma_y)+M\sigma_z}{\sqrt{M^2+\alpha^2k^2}}\right). \tag{11}$$

Then, along the lines of [23, 28], first we calculate the anomalous Hall conductivity σ^0_{xy}, which corresponds to the nonvertex correction at zero temperature and zero frequency, based on Kubo's formula

$$\sigma^0_{xy} = \frac{e^2\hbar}{2\pi V}\sum_{\mathbf{k}}\mathrm{Tr}\,[G^a(\varepsilon_\mathrm{f},\mathbf{k})v_xG^r(\varepsilon_\mathrm{f},\mathbf{k})v_y] \tag{12}$$

where e is the charge of the electron. ε_f is the Fermi energy, and v_x and v_y have been obtained in equations (5) and (6), respectively. Applying $\lim_{V\to\infty}(1/V)\sum_{\mathbf{k}} \to \int \mathrm{d}^2\mathbf{k}/(2\pi)^2$, the summation in equation (12) can be turned into the integral. Then we introduce another new dimensionless integral variable: $q = \sqrt{\alpha^2k^2+M^2}/\varepsilon_\mathrm{f}$, and subsequently we decompose the denominator in equation (12) coming from the product of the denominator of the retarded and advanced Green's functions, $(\varepsilon_\mathrm{f}-\varepsilon_\lambda-\mathrm{i}\hbar/2\tau)(\varepsilon_\mathrm{f}-\varepsilon_{\lambda'}+\mathrm{i}\hbar/2\tau)$, into the form: $(\varepsilon_\mathrm{f}/a^2)^2(q-q_1)(q-q_2)(q-q_3)(q-q_4)$, where q_i ($i=1,2,3,4$) represent the poles. We assume that the Fermi energy is the largest energy scale, i.e. $\varepsilon_\mathrm{f}\gg\hbar/\tau, \Delta$. Within the accuracy of $\mathrm{O}(b,r)$ and in a up to second order, the four poles can be expressed as:

$$\begin{aligned} q_1 &= -\lambda\frac{a^2}{2}+b-\mathrm{i}\frac{r}{4};\\ q_2 &= -\lambda\frac{a^2}{2}-b+\mathrm{i}\frac{r}{4};\\ q_3 &= -\lambda'\frac{a^2}{2}+b+\mathrm{i}\frac{r}{4};\\ q_4 &= -\lambda'\frac{a^2}{2}-b-\mathrm{i}\frac{r}{4} \end{aligned} \tag{13}$$

where the dimensionless constants $a = \alpha k_\mathrm{f}/\varepsilon_\mathrm{f}$, $b = \sqrt{\alpha^2k_\mathrm{f}^2+M^2}/\varepsilon_\mathrm{f}$, and $r = a^2\hbar/(b\varepsilon_\mathrm{f}\tau)$ are introduced, in the assumption of the largest Fermi energy scale that $a, b, r \ll 1$. Now, based on these preparations, we can calculate the Hall conductivity with the help of residue theorem and obtain the Hall conductivity without vertex correction:

$$\sigma^0_{xy} = -4e^2\hbar\nu\frac{\alpha^2}{\hbar^2}M\frac{\tau^2/\hbar^2}{1+(2\Delta\tau/\hbar)^2} \tag{14}$$

where $2\Delta = 2\sqrt{M^2+\alpha^2k_\mathrm{f}^2}$ denotes the splitting energy between the two separated branches in the presence of spin–orbit interaction and magnetization at the Fermi surface, when the two branches are both occupied, and k_f is the Fermi wavevector. Clearly, σ^0_{xy} in equation (14) is the same as the conclusion derived by Inoue *et al* [24] for isotropic spin-dependent impurities in the approximation $M \gg \alpha k_\mathrm{f}$. By simple calculation, we can also find that the conclusion is the same as in the nonmagnetic impurities case.

3.2. Vertex correction

Furthermore, we calculate the vertex correction σ^L_{xy} to the anomalous Hall conductivity, which corresponds to the ladder diagram, and the sum of ladder diagrams is expressed as

$$\sigma^L_{xy} = \frac{e^2\hbar}{2\pi V}\sum_{\mathbf{k}}\mathrm{Tr}[G^a(\varepsilon_\mathrm{f},\mathbf{k})\widetilde{v}_xG^r(\varepsilon_\mathrm{f},\mathbf{k})v_y] \tag{15}$$

where $\widetilde{v}_x$ is the vertex correction to the v_x. Also, the equation for the vertex $\widetilde{v}_x$ can be expressed in the following form in the ladder approximation in the presence of short-ranged magnetic impurities:

$$\begin{aligned}\widetilde{v}_x = {} & \frac{n_\mathrm{i}u^2}{V}\sum_{\mathbf{k}}\int \mathrm{d}\theta\,\mathrm{d}\phi\frac{1}{4\pi}\sin\theta\begin{pmatrix}\gamma\cos\theta & \sin\theta\mathrm{e}^{-\mathrm{i}\phi}\\ \sin\theta\mathrm{e}^{\mathrm{i}\phi} & -\gamma\cos\theta\end{pmatrix}\\ & \times G^a(\varepsilon_\mathrm{f},\mathbf{k})[v_x+\widetilde{v}_x]G^r(\varepsilon_\mathrm{f},\mathbf{k})\\ & \times\begin{pmatrix}\gamma\cos\theta & \sin\theta\mathrm{e}^{-\mathrm{i}\phi}\\ \sin\theta\mathrm{e}^{\mathrm{i}\phi} & -\gamma\cos\theta\end{pmatrix}.\end{aligned} \tag{16}$$

Also, we can find that the vertex $\widetilde{v}_x$ should be independent of momentum $\mathbf{k}$. Then we look for a solution of equation (16) in the form

$$\widetilde{v}_x = \begin{pmatrix}(\widetilde{v}_x)_{\uparrow\uparrow} & (\widetilde{v}_x)_{\uparrow\downarrow}\\ (\widetilde{v}_x)_{\downarrow\uparrow} & (\widetilde{v}_x)_{\downarrow\downarrow}\end{pmatrix}. \tag{17}$$

Before the calculation of the anisotropy magnetic impurity case, for comparison we would like to review the nonmagnetic condition in brief, with a δ-function-shaped (weak, short-ranged) impurity scattering potential, and the disorder average $\langle V_\mathrm{m}(\mathbf{r})V_\mathrm{m}(\mathbf{r}')\rangle = n_\mathrm{i}u^2\delta(\mathbf{r}-\mathbf{r}')$, which induces the momentum relaxation time $\hbar/\tau = n_\mathrm{i}u^2 2\pi\nu$. Similarly to the magnetic case, the vertex $\widetilde{v}^0_x$ consists of four components: $(\widetilde{v}^0_x)_{\uparrow\uparrow}$, $(\widetilde{v}^0_x)_{\uparrow\downarrow}$, $(\widetilde{v}^0_x)_{\downarrow\uparrow}$, $(\widetilde{v}^0_x)_{\downarrow\downarrow}$, respectively. Also, they satisfy the diagram equation within the impurity ladder approximation:

$$\widetilde{v}^0_x = n_\mathrm{i}u^2\int\frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2}G^a(\varepsilon_\mathrm{f},\mathbf{k})[v_x+\widetilde{v}^0_x]G^r(\varepsilon_\mathrm{f},\mathbf{k}). \tag{18}$$

After simple calculation, in the limit of a large Fermi energy level $\varepsilon_f \gg \Delta, \hbar/\tau$, we can obtain:

$$
\begin{aligned}
(\widetilde{v}_x^0)_{\uparrow\uparrow} &= \frac{1}{2}\left((\widetilde{v}_x^0)_{\uparrow\uparrow} + (\widetilde{v}_x^0)_{\downarrow\downarrow}\right) + \frac{A^0}{2}((\widetilde{v}_x^0)_{\uparrow\uparrow} - (\widetilde{v}_x^0)_{\downarrow\downarrow});\\
(\widetilde{v}_x^0)_{\downarrow\downarrow} &= \frac{1}{2}((\widetilde{v}_x^0)_{\uparrow\uparrow} + (\widetilde{v}_x^0)_{\downarrow\downarrow}) - \frac{A^0}{2}((\widetilde{v}_x^0)_{\uparrow\uparrow} - (\widetilde{v}_x^0)_{\downarrow\downarrow});\\
(\widetilde{v}_x^0)_{\uparrow\downarrow} &= \frac{B^0}{2} + \frac{C^0}{2}(\widetilde{v}_x^0)_{\uparrow\downarrow};\\
(\widetilde{v}_x^0)_{\downarrow\uparrow} &= -\frac{B^0}{2} + \frac{C^0}{2}(\widetilde{v}_x^0)_{\downarrow\uparrow}
\end{aligned}
\tag{19}
$$

where

$$
\begin{aligned}
A^0 &= \frac{M^2}{\Delta^2} + \frac{1}{1+(2\Delta\tau/\hbar)^2} - \frac{M^2}{(1+(2\Delta\tau/\hbar)^2)\Delta^2};\\
B^0 &= -\mathrm{i}\frac{\alpha}{\hbar}(1+\frac{M^2}{\Delta^2})(1-\frac{1}{1+(2\Delta\tau/\hbar)^2});\\
C^0 &= 1 - \frac{M^2}{\Delta^2} + \frac{1}{1+(2\Delta\tau/\hbar)^2} + \frac{M^2}{(1+(2\Delta\tau/\hbar)^2)\Delta^2}.
\end{aligned}
\tag{20}
$$

From the first lines of the above equations (19), we find $(\widetilde{v}_x^0)_{\uparrow\uparrow} = (\widetilde{v}_x^0)_{\downarrow\downarrow}$, which have no contribution to the vertex correction of the Hall conductivity. From the last two lines, we can obtain

$$
(\widetilde{v}_x^0)_{\uparrow\downarrow} = -(\widetilde{v}_x^0)_{\downarrow\uparrow} = -\mathrm{i}\frac{\alpha}{\hbar}. \tag{21}
$$

Then, with the help of equation (15), we can obtain the results directly,

$$
\sigma_{xy}^{L0} = 4e^2\hbar\nu\frac{\alpha^2}{\hbar^2}M\frac{\tau^2/\hbar^2}{1+(2\Delta\tau/\hbar)^2} \tag{22}
$$

and finally, for the nonmagnetic impurities case, the total anomalous Hall conductivity is

$$
\sigma_{xy}^n = \sigma_{xy}^0 + \sigma_{xy}^{L0} = 0. \tag{23}
$$

Clearly, because of the presence of vertex correction, the two contributions to the anomalous Hall conductivity cancel each other out, and finally the Hall conductivity is vanishing. Just as Smit [6] pointed out, the intrinsic effect of the applied electric field is completely canceled out by collision, and there is no stationary anomalous Hall effect in the disordered sample.

Now, we turn to calculate the vertex in the presence of magnetic impurities. Using the same method, substituting the expression (equation (17)) into equation (16) and with the help of equation (8), we find, in the limit of large Fermi energy level $\varepsilon_f \gg \Delta, \hbar/\tau$, that

$$
\begin{aligned}
(\widetilde{v}_x)_{\uparrow\uparrow} &= \frac{1}{2}\left((\widetilde{v}_x)_{\uparrow\uparrow} + (\widetilde{v}_x)_{\downarrow\downarrow}\right) + \frac{A}{2}((\widetilde{v}_x)_{\uparrow\uparrow} - (\widetilde{v}_x)_{\downarrow\downarrow});\\
(\widetilde{v}_x)_{\downarrow\downarrow} &= \frac{1}{2}((\widetilde{v}_x)_{\uparrow\uparrow} + (\widetilde{v}_x)_{\downarrow\downarrow}) - \frac{A}{2}((\widetilde{v}_x)_{\uparrow\uparrow} - (\widetilde{v}_x)_{\downarrow\downarrow});\\
(\widetilde{v}_x)_{\uparrow\downarrow} &= \frac{B}{2} + \frac{C}{2}(\widetilde{v}_x)_{\uparrow\downarrow};\\
(\widetilde{v}_x)_{\downarrow\uparrow} &= -\frac{B}{2} + \frac{C}{2}(\widetilde{v}_x)_{\downarrow\uparrow}
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
A &= \frac{\gamma^2-2}{\gamma^2+2}\left(\frac{M^2}{\Delta^2} + \frac{1}{1+(2\Delta\tau/\hbar)^2} - \frac{M^2}{(1+(2\Delta\tau/\hbar)^2)\Delta^2}\right);\\
B &= \frac{\mathrm{i}\alpha\gamma^2/\hbar}{\gamma^2+2}\left(1+\frac{M^2}{\Delta^2}\right)\left(1-\frac{1}{1+(2\Delta\tau/\hbar)^2}\right);\\
C &= \frac{-\gamma^2}{\gamma^2+2}\left(1 - \frac{M^2}{\Delta^2} + \frac{1}{1+(2\Delta\tau/\hbar)^2} + \frac{M^2}{(1+(2\Delta\tau/\hbar)^2)\Delta^2}\right).
\end{aligned}
\tag{25}
$$

From the first two lines of equations (24) we find $(\widetilde{v}_x)_{\uparrow\uparrow} = (\widetilde{v}_x)_{\downarrow\downarrow}$, similar to the nonmagnetic condition. They have no contribution to the vertex correction, and we can set them to zero for simplification. From the last two lines, we can obtain

$$
(\widetilde{v}_x)_{\uparrow\downarrow} = -(\widetilde{v}_x)_{\downarrow\uparrow} = \frac{B}{2-C}. \tag{26}
$$

Then, substituting the above results into equation (15), the vertex correction to the anomalous Hall conductivity is evaluated directly, expressed as

$$
\sigma_{xy}^L = 4\mathrm{i}e^2\hbar\nu\frac{\alpha}{\hbar}M\frac{\tau^2/\hbar^2}{1+(2\Delta\tau/\hbar)^2}(\widetilde{v}_x)_{\uparrow\downarrow} \tag{27}
$$

and the total anomalous Hall conductivity is

$$
\begin{aligned}
\sigma_{xy} &= \sigma_{xy}^0 + \sigma_{xy}^L\\
&= \left(1 + \frac{\gamma^2(1+\frac{M^2}{\Delta^2})(1-\frac{1}{1+(2\Delta\tau/\hbar)^2})}{\gamma^2+2+\gamma^2(\frac{\Delta^2-M^2}{\Delta^2}+\frac{\Delta^2+M^2}{(1+(2\Delta\tau/\hbar)^2)\Delta^2})}\right)\sigma_{xy}^0.
\end{aligned}
\tag{28}
$$

4. Discussion and summary

In the limit of $\varepsilon_f \gg \hbar/\tau, \Delta$, the anomalous Hall conductivity of the above result (equation (28)) can be simplified further by assuming that $M \gg \alpha k_f$ and $\Delta\tau/\hbar \gg 1$:

$$
\sigma_{xy} = \left(1 + \frac{2\gamma^2}{\gamma^2+2}\right)\sigma_{xy}^0. \tag{29}
$$

We can easily see from equation (29) that the expression for σ_{xy} is odd with respect to magnetization $\mathbf{M}$, as expected, and that $\sigma_{xy} = 0$ at the limit $\mathbf{M} = 0$. Now, the anomalous Hall conductivity is no longer zero in the presence of the magnetization $\mathbf{M}$, even when the vertex correction is taken into account, which is different from the nonmagnetic and isotropic spin-dependent impurity condition [24]. This is not difficult to understand from two aspects. First, it is easy to find that, in the calculation of the vertex correction, the B^0 and B corresponding to the nonmagnetic and magnetic impurity conditions, respectively, have opposite sign, which is due to the matrix elements for the magnetic potential including the term $\langle\lambda|\sigma_z|\lambda\rangle$, while in the nonmagnetic case the potential is spin-independent and has the opposite sign. This is just as explained by Inoue [24] for the nonvanishing spin Hall conductivity. Second, from the viewpoint of impurity scattering, when the scattering is spin-dependent, up and down spin electrons are scattered in opposite directions, resulting in spin-up and spin-down charge currents along the direction perpendicular to the external electric field. In normal semiconductors, the electrons are not spin polarized, so that an equal number of electrons with up and down spin are scattered in opposite directions, respectively. Then there will be no Hall voltage, but spin accumulation exists, which can be used to explain the nonvanishing spin Hall conductivity in magnetic impurities [24, 25]. Meanwhile, in ferromagnets the intrinsic spin imbalance makes the charge in the two scattering directions different and a Hall voltage proportional to the magnetization is induced. But, if the scattering is spin-independent, then there will be no such properties and it is not surprising that we obtain vanishing anomalous or spin Hall conductivities.

With the experimental parameters, the order of magnitude of the anomalous Hall conductivities can be estimated for different anisotropy γ. We take the electron's effective mass as $m = 0.05\ m_e$, the impurity density as $n_i = 1.2 \times 10^{10}$ cm^{-2}, and the impurity scattering strength as $u = 0.2 \times 10^{-12}$ meV cm, which ensures that the relaxation time is $\tau \sim$ 1 ps. Also, the electron density is taken as $n = 1.4 \times 10^{12}$ cm^{-2}, which allows us to estimate the Fermi wavevector $k_f = \sqrt{2\pi n} = 3 \times 10^6$ cm^{-1}. In addition, the spin–orbit coupling coefficient α is varied from 1.0×10^{-11} to 1.0×10^{-10} eV m, which can be modulated by an external gate, and the magnitude of magnetization $\mathbf{M}$ is taken as $M/\varepsilon_f \sim 0.6$. Here we only consider the condition that the Fermi energy is the largest energy scale and that the magnetization magnitude $M > \alpha k_f$. With these parameters, we can find that $\Delta\tau/\hbar \gg 1$ and, for different anisotropy γ, the anomalous Hall conductivities are varying on the order of 0.001–0.1e^2/h for different spin–orbit coupling coefficients. In addition, the anomalous Hall conductivity is anisotropy dependent.

In summary, on the basis of Kubo's linear response theory, in the ladder approximation the vertex correction to the anomalous Hall conductivity is evaluated by taking into account the weak, short-ranged nonmagnetic and anisotropic magnetic impurities. Absolutely contrary conclusions are obtained for the two cases. In the presence of nonmagnetic impurities, the anomalous Hall conductivity from the nonvertex component and the vertex correction cancel each other, and the total anomalous Hall conductivity is zero. Different from the nonmagnetic case, in the presence of magnetic impurities the two contributions to the anomalous Hall conductivity cannot cancel each other, and the anomalous Hall conductivity is nonvanishing. In the assumption of $\varepsilon_f \gg \hbar/\tau, \Delta$, the anomalous Hall conductivity not only depends on the magnetization M, the momentum relaxation time τ, and the spin–orbit coupling constant α, but is also sensitive to the anisotropy γ. As $M \to 0$, the anomalous Hall conductivity σ_{xy} goes to zero, consistent with the facts, but spin accumulation still exists, and the so-called spin Hall conductivity is nonzero, then the question turns back to research on the spin Hall effect.

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